logical, good results should be obtainable from equations (1) with $m_+ = m_- = \sqrt{3}$.

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AN ENGINEERING APPROXIMATION FOR RESISTANCES OF CERTAIN TWO-DIMENSIONAL CONDUCTORS

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MODEL DESCRIPTION

The model studied was a two-dimensional conductor of thickness l containing a cut of depth m and width n (cf. sketch in Fig. 1). The conductivity was taken to be constant. The problem, from the viewpoint of the designer, may be stated: Predict the length L' of an equivalent conductor of uniform thickness l having the same gross conduction properties as a conductor of length L containing a cut such as depicted in Fig. 1. In order to obtain the solution to this problem, one may consider either of the following cases:

1. Insulated surfaces, flow along the conductor.
2. Constant-potential surfaces, flow across the conductor.

The constant-potential and flow lines of the first case are respectively the flow and constant-potential lines of the second case. The cut increases the length of the equivalent conductor, in the first case, by increasing the resistance to flow and, in the second case, by decreasing the resistance to flow. For convenience, the discussion during the bulk of the paper is in terms of the first case.

ANALYTICAL STUDIES

Analyses of potential flows in systems with polygonal boundaries are facilitated by use of the Schwarz–Christoffel transformation [1, p. 445]. Using this method, Schofield [2] finds for the conductor being considered

$$\frac{L'}{l} = \frac{L-n}{l} + \frac{n}{l-m} + \frac{X}{l} \tag{1}$$

$$\frac{X}{l} = \frac{2}{\pi}\ln\left[\frac{2\,\mathrm{sn}\alpha}{\mathrm{cn}\alpha\,\mathrm{dn}\alpha\,\mathrm{sn}2\alpha}\,\frac{\Theta(0)}{\Theta(2\alpha)}\right] + \frac{n}{l}\left(1-\frac{\alpha}{K}\right) - \frac{n}{l-m} \tag{2}$$

where the constants α and K are to be determined by solving simultaneously

$$\frac{k^2\,\mathrm{sn}\alpha\,\mathrm{cn}\alpha}{\mathrm{dn}\alpha} - Z(\alpha) = \frac{\pi}{4K}\frac{n}{l} \tag{3}$$

$$\frac{k^2\,\mathrm{sn}\alpha\,\mathrm{cn}\alpha}{\mathrm{dn}\alpha} - Z(\alpha) = \frac{\pi}{2K'}\left(\frac{\alpha}{K} - \frac{m}{l}\right) \tag{4}$$

(see, e.g. [3] for a discussion of elliptic integrals which is brief and yet is adequate for the present discussion). The first two terms appearing in the right-hand side of equation (1), i.e. $(L-n)/l$ and $n/(l-m)$, represent respectively the resistances of the parts with thicknesses l and $l-m$ to one-dimensional flows; the term X/l represents the additional resistance due to the fact that the flow paths are curved in the vicinity of the junctions of the parts with differing thicknesses. The probability that a design engineer would take the time required to solve simultaneously equations (3) and (4) appears to be small. Hence, a convenient approximate solution to equations (2–4) is sought.

A useful guide in the search for this approximate solution is provided by the physical interpretation of the term X/l—"the additional resistance due to the fact that the flow paths are curved". Curvature of the flow paths is due primarily to the fact that m/l is non-zero. Hence one is motivated to examine the term X/l for the limiting

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cases of narrow and wide cuts. As shown already by Lees, if $n/l = 0$, then [4]

$$\frac{X}{l} = \frac{4}{\pi} \ln \sec \frac{\pi}{2} \frac{m}{l}. \tag{5}$$

Whereas, if $n/l \to \infty$, then [5]

$$\frac{X}{l} = \frac{4}{\pi} \left[\ln \frac{l^2 - (l - m)^2}{4l(l - m)} + \frac{l^2 + (l - m)^2}{2l(l - m)} \ln \frac{l + (l - m)}{l - (l - m)} \right]. \tag{6}$$

For small and intermediate values of m/l, these two limiting cases are compared in Fig. 1. For m/l near unity, i.e. for $(l - m)/l \ll 1$,

$$\frac{X}{l} \approx \frac{4}{\pi} \left(\ln \frac{l}{l - m} - 0{\cdot}386 \right) \text{ if } \frac{n}{l} = 0 \tag{5a}$$

and

$$\frac{X}{l} \approx \frac{4}{\pi} \left(\ln \frac{l}{l - m} - 0{\cdot}451 \right) \text{ if } \frac{n}{l} = \infty. \tag{5b}$$

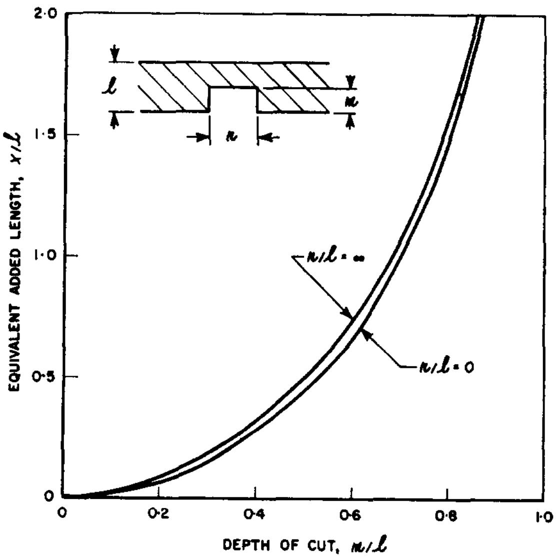

FIG. 1. Equivalent added length as function of depth of cut for limiting values of width of cut.

It is seen that the difference between the equivalent added length predicted for these two limiting values of n/l is not great. Hence, one might use, as an engineering approximation, the simpler of equations (5) and (6) and write for all values of n/l,

$$\frac{L'}{l} \approx \frac{L - n}{l} + \frac{n}{l - m} + \frac{4}{\pi} \ln \sec \frac{\pi}{2} \frac{m}{l}. \tag{7}$$

Since all three terms on the right-hand side of equation (7) are inherently positive quantities, the percentage difference between the two curves of Fig. 1 represents a conservative upper limit to the percentage error introduced approximating equations (1–4) by equation (7).

It is to be kept in mind that all of the equations presented here are applicable only to cases in which the length is sufficiently great so that the flow paths at the ends are nearly parallel to the bounding surfaces. However, Schofield [2] finds, e.g. that, for $n/l = 0$ and $m/l = \frac{3}{4}$, if the distance from the cut to the end of the conductor is greater than the thickness of the conductor, then the error made using an equation derived for very long conductors is of the order of a tenth of a per cent.

ANALOG STUDIES

A comparison of the predictions of equation (7) with results obtained by an independent method are desirable in order to establish the usefulness of this equation. The method chosen for this purpose is the analog method perfected by Hele-Shaw and Hay [6] and used extensively in recent years by Moore [7]; the two-dimensional flow paths are represented by the stream lines realized in the case of viscous fluid flow between parallel plates [one might have chosen alternatively the electrical analog method using conducting (Teledeltos) paper]. Resistances measured in the analog study ($0{\cdot}166 \leqslant n/l \leqslant 0{\cdot}450$, $0{\cdot}505 \leqslant m/l \leqslant 0{\cdot}975$, and $4{\cdot}08 \leqslant L/l \leqslant 4{\cdot}72$) agreed with the predictions of equation (7) within the limits of probable experimental error.

DISCUSSION

The author recalls seeing a proposal to simulate a conduction path with abrupt changes in conductivity by a conduction path with constant conductivity and abrupt changes in cross-sectional area. An examination of equation (1) reveals that a necessary condition for this simulation is that the last term (the term representing additional resistance due to the fact that the flow paths are curved) either is negligible in comparison with the other terms or is taken into account properly.

Although the study described in this paper was motivated by an interest in heat-conduction problems, the results are applicable to any two-dimensional phenomena described by the Laplace equation. For example, the results are applicable also to flow of an electrical current either along or across a conducting strip of uniform thickness and with boundary conditions of the type considered in this study. Note that, by combining, with symmetry about a horizontal plane, models of the type shown in Fig. 1, certain slightly more complicated conductor geometries may be formed.

The realized agreement between the prediction of equation (7) and the results of the analog study establishes the usefulness of equation (7) as an engineering approximation. It is hoped that this equation provides a simple approximate method for handling some of the complicated geometrical effects encountered frequently, e.g. in current heat-transfer design calculations.

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HEAT-TRANSFER MEASUREMENTS DURING DROPWISE CONDENSATION OF STEAM

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THE greatly enhanced heat-transfer coefficient obtainable in the presence of dropwise condensation has long engaged the interest of research workers. Hitherto, lack of precise measurements has hindered basic studies of the mechanism. The present investigation had, as its original aim, the provision of precise data relating to heat-flux and steamside temperature difference. It is briefly reported here in order to render available the *results*. These may be briefly summarized thus:

For each of four different promoters, runs were made using three different effective plate heights. Observations of hitherto unexcelled consistency were obtained and found reproducible on different days. These results are thought to have enhanced precision and, in particular, their relation to earlier work supports the view that the effect of "non-condensables" has been avoided. Differences between promoters were clearly established.

Thermocouples, accurately located and spaced through the test plates served to measure:

(1) by extrapolation—the "mean" surface temperature at a known point on the condensing surface;
(2) from the temperature gradient—the heat flux.

Each of two copper test plates was 0·5 in thick and the dimensions of the condensing surfaces were:

	Horizontal	*Vertical*
Plate 1	2·750 in	2·44 in
Plate 2	0·875 in	5·00 in

Using Plate 1, the measurements were made at a point 1·12 in below the top of the condensing surface. Using Plate 2, measurements were made at depths of 1 in and 4 in from the top of the condensing surface. The plates, in all cases were vertical and the steam pressure was approximately 1·03 atm. In all three cases the measuring point was laterally central.

The promoters used were:

(1) Dioctadecyl disulphide [$C_{18}H_{37}SSC_{18}H_{37}$]
(2) "No. 1 Amine"† [chiefly octadecylamine $C_{18}H_{37}NH_2$]
(3) Di-S-octadecyl 00 − 1, 10-decanedixanthate [$C_{18}H_{37}SSCO(CH_2)_{10}OCSSC_{18}H_{37}$]
(4) Dodecanetris (ethanethio) silane [$C_{12}H_{25}Si(SC_2H_5)_3$]

The graphs presented (see Fig. 1) relate to the condensation of clean steam, free from "non-condensable" gases, on vertical copper surfaces operating under steady conditions a few hours after promotion.

It was found that when a plate had been newly promoted, the surface temperature (after the removal of "non-condensables") increased for some time. In most cases an interval of at least 4 h was required to attain a steady and reproducible value. The increase in surface temperature during this interval was between 1 and 2 degC. No further change was detected over the duration of the test. Presumably, during the preliminary interval "excess" promoter is removed by the condensate.

Fig. 1 indicates that for all four promoters, the steam-to-surface temperature difference is an approximately linear function of heat flux over the range of heat flux investigated. It was also found that the position of the measuring point did not affect the results over the range used (1 in to 4 in from the top of the condensing surface).

It may also be seen that although promoters 1 and 2 give very nearly the same results, 3, and particularly 4, give results that are evidently different.

† "No. 1 Amine" is a commercial product of Houseman Thomson and Co. Ltd., and is used as a corrosion inhibitor.